

Title: On graphs representable by words

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Abstract: A simple graph $G=(V,E)$ is (word-)representable if there exists a word W over the alphabet V such that any two distinct letters x and y alternate in W if and only if (x,y) is an edge in E . If W is k -uniform (each letter of W occurs exactly k times in it) then G is called k -representable. It is known that a graph is representable if and only if it is k -representable for some k . Minimum k for which a representable graph G is k -representable is called its representation number.

Representable graphs appeared first in algebra, in study of the Perkins semigroup which has played a central role in semigroup theory since 1960, particularly as a source of examples and counterexamples. However, these graphs have connections to robotic scheduling and they are interesting from combinatorial and graph theoretical point of view (for example, representable graphs are a generalization of circle graphs, which are exactly 2-representable graphs).

Some of questions one can ask about representable graphs are as follows. Are all graphs representable? How do we characterize those graphs that are (non-)representable? How many representable graphs are there? How large the representation number can be for a graph on n nodes?

In this talk, we will go through these and some other questions stating what progress has been made in answering them. In particular, we will see that a graph is representable if and only if it admits a so called semi-transitive orientation. This allows us to prove a number of results about representable graphs, not the least that 3-colorable graphs are representable. We also prove that the representation number of a graph on n nodes is at most n , from which one concludes that the recognition problem for representable graphs is in NP. This bound is tight up to a constant factor, as there are graphs whose representation number is $n/2$.